Nonlinear evolution of electron two-stream instability:
Two-dimensional particle simulation

Takayuki Umeda1, Yoshiharu Omura2, Taketoshi Miyake3, and Hiroshi Matsumoto2,

1 Institute of Geophysical and Planetary Physics, University of California, Los Angeles, California 90095-1567, USA
2 Research Institute for Sustainable Humanosphere, Kyoto University, Uji, Kyoto 611-0011, Japan
3 Toyama Prefectural University, Kougai, Toyama 930-0398, Japan

We study nonlinear evolution of electron two-stream instabilities in a two-dimensional system. Electron two-stream and bump-on-tail instabilities are considered to be the most probable generation mechanisms of electrostatic solitary waves/electron holes observed in various regions of the Earth’s magnetosphere. To obtain detailed understanding on the nonlinear evolution of two-stream instability, we performed two-dimensional electromagnetic particle simulations with various sets of the electron cyclotron frequencies and the initial electron thermal velocities. We found that the nonlinear evolution falls into four groups depending on these parameters.

1. Introduction

Electrostatic Solitary Waves (ESW) were first observed by the GEOTAIL spacecraft in the Earth’s magnetotail [Matsumoto et al., 1994]. ESW are bipolar electric pulses longitudinal to the ambient magnetic field. ESW can be modeled as electron phase-space density holes which are Bernstein-Greene-Kruskal (BGK) modes, i.e., one-dimensional equilibrium solutions to the time-independent Vlasov-Poisson equations [Bernstein et al., 1957; Krasovsky et al., 2003; Muschietti et al., 1999]. The previous Particle-In-Cell (PIC) simulations demonstrated that one-dimensional electron holes in the PSBL region are generated through longtime nonlinear evolution of an electron bump-on-tail instability [Omura et al., 1994, 1996; Miyake et al. 1998; Umeda et al., 2002, 2004].

Recently, electrostatic solitary waves are observed in various regions of the Earth’s magnetosphere. Spatial structures and stability of multi-dimensional electron holes observed by the FAST spacecraft in the auroral region [Ergun et al., 1998] are issues of recent computer simulations. According to the previous multi-dimensional simulations of electron beam instabilities, electron holes are unstable under a very weak ambient magnetic field such that $\Omega_e < \omega_i$ [Miyake et al., 1998; Muschietti et al., 2000]. When the ambient magnetic field is strong such that $\Omega_e \gg \Pi_e$, electron holes are also unstable to excite electrostatic whistler waves [Goldman et al., 1999; Oppenheim et al., 1999]. Under a weaker ambient magnetic field such that $\Omega_e \sim \Pi_e$, electron holes are stable and one-dimensional electron holes are formed through the coalescence process [Muschietti et al., 1999; Miyake et al., 2000].

The purpose of the present study is to obtain further understanding on the nonlinear evolution of electron two-stream instability in a two-dimensional system. We performed two-dimensional particle simulations with different initial thermal velocities of two electron beams and different magnitudes of ambient magnetic field. We focus on nonlinear processes at a relatively early stage in the timescale of $\Pi_e t \sim 1000$, and therefore the ion dynamics is neglected.

2. Model and parameters

Computer simulations were performed with a two-dimensional electromagnetic particle-in-cell code based on Kyoto university ElectroMagnetic Particle cOde (KEMPO) [Omura and Matsumoto, 1993] and a new charge conservation method for solving the continuity equation for charge [Umeda et al., 2003]. The simulation system is taken in the $x-y$ plane with $N_x \times N_y = 256 \times 256$ grid points. In both $x$ and $y$ directions we imposed periodic boundary conditions for both fields and particles. The ambient magnetic field is taken in the $x$ direction.

We assumed counter-streaming two electron beams with an equal density $0.5n_0$ and equal initial thermal velocity $V_t$. The two electron beams drift along the ambient magnetic field with the drift velocity $V_d$ and $-V_d$. We used 64 particles per cell for each electron beam component, while ions are assumed to be immobile background to exclude the effect of the ion dynamics. Although the number of superparticles per cell is not so large, we adopted the second-order (i.e., triangular) particle shape for solving particle velocities and current densities. Therefore, numerical heating of particle energy due to enhanced thermal fluctuation is reduced substantially. We varied the initial thermal velocity from $V_t/V_d = 0.5$ to 0.25, and the electron cyclotron frequency from $\Omega_e/\Pi_e = 0.5$ to 10 as shown in Table 1. To increase computation efficiency we assumed a reduced light speed, $c = 10V_d$. The grid spacings $\Delta x$ and $\Delta y$ are equal to $0.5V_d/\Pi_e$, and the time step is equal to $\Pi_e \Delta t = 0.025$.

3. Simulation results

As shown in Table 1, we performed 30 runs with various sets of $V_t/V_d$ and $\Omega_e/\Pi_e$. We found that nonlinear evolution of electron two-stream instability in the two-dimensional system falls into 4 categories as summarized in Table 1. Here we describe the main features of electrostatic potential structures as seen from these simulations.

As seen in the previous simulations [Miyake et al., 1998, 2000; Muschietti et al., 1999, 2000; Singh et al., 2001], nonlinear evolution of electron two-stream instability is controlled by the magnitude of ambient magnetic field. When
the bounce frequency of electrons trapped by potential well of an electron hole, \( \omega_b \), is larger than the electron cyclotron frequency, electron holes are unstable to diffuse at the saturation stage, and there exist no potential structure at the nonlinear stage. This result is in good agreement with Muschietti et al. [2000]. However, under a slightly stronger ambient magnetic field such that \( \omega_b < \Omega_e < 2\omega_b \), we found the formation of stable two-dimensional electron holes isolated in both directions parallel and perpendicular to the ambient magnetic field. In the present simulations, two-dimensional electron holes persist for more than several thousands of electron plasma frequency, but the amplitude of two-dimensional electron holes gradually decreases. When the electron cyclotron frequency is larger than the electron plasma frequency, we found that the stability of electron holes is controlled by their amplitudes. In the present simulations, nonlinear evolution of two-stream instability shows almost the same feature for \( \Omega_e/\Pi_e > 2.0 \). We found that electrostatic whistler waves are excited from electron holes for \( V_t/V_d \leq 0.4 \) as seen in the previous simulations [Goldman et al., 1999; Oppenheim et al., 1999]. On the other hand, stable one-dimensional electron holes are formed for \( V_t/V_d > 0.4 \). This result suggests that \( \Omega_e/\Pi_e > 1.0 \) is a necessary but not a sufficient condition for the excitation of electrostatic whistler waves.

When the potential energy of electron holes is smaller than or is comparable to the thermal energy of electrons, the two-stream instability develops to form one-dimensional electron holes. We confirmed that electron holes are stable for a long time (\( \Pi_e t > 5000 \)) as seen in the previous simulations [Miyake et al., 1998, 2000; Muschietti et al., 1999]. The transition process of spatial structures of electron holes through parallel and perpendicular coalescence of electron holes is studied by Miyake et al. [1998, 2000]. On the other hand, when the potential energy of electron holes is larger than the thermal energy of electrons, electron holes decay into electrostatic whistler waves. The electrostatic whistler modes are excited at an earlier stage as the initial electron thermal velocity becomes smaller. The present simulation results showed that the amplitude of electron holes is more essential for the excitation of electrostatic whistler waves.

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**References**


