Finite amplitude Alfvén wave in a relativistic electron-positron plasma

T. Hada¹, S. Matsukiyo¹, V. Muñoz², and M. Ikeda¹

¹ Department of Earth System Science and Technology, Kyushu University, Fukuoka 816-8580, Japan
² Departamento de Física, Facultad de Ciencias, Universidad de Chile, Casilla 653, Santiago, Chile

Propagation of a finite amplitude, parallel, circularly polarized Alfvén wave in a relativistic electron-positron plasma is studied theoretically and by means of numerical simulations. In the linear dispersion relation, due to relativistic effects, there is no low frequency cutoff for the electromagnetic branch, and there appears a critical wave number above which the Alfvén wave ceases to exist. Besides, for each frequency in the Alfvén branch, two additional forward propagating modes exist with equal frequency. In order to study the peculiarities of the linear dispersion relations, several approaches are being performed, namely, a fluid simulation based on a rationalized Runge-Kutta algorithm, an electromagnetic full particle code, as well as numerical analysis of oblique propagations. We report early results with these approaches.

1. Introduction

Electron-positron plasmas are different from electron-ion plasmas, because there are no high or low natural frequency scales. Such plasmas are found in pulsar magnetospheres, models of primitive Universe, active galactic nuclei jets, and laboratory and tokamak plasmas. Relativistic effects are expected to play an important role in several of these systems. Understanding interactions between waves and relativistic electron-positron plasmas is relevant to proposed pulsar emission mechanisms, and may give insight into structure formation in the early Universe.

Therefore, wave propagation in relativistic electron-positron plasmas has been the subject of many studies, either in the fluid or the kinetic treatments: linear waves [1], nonlinear waves [2], and nonlinear decays [3, 4, 5].

In this article we deal with an Alfvén wave propagating along a constant magnetic field in a pair plasma. When fully relativistic effects are considered in the particle motion, the dispersion relation exhibits unique features which, to our knowledge, have not been discussed before. We then outline the numerical strategies we are currently considering to examine the consequences of such features.

2. Linear dispersion relation

The system is described by Maxwell’s equations, a continuity equation for the density, and the fluid force equation

\[
\left( \frac{\partial}{\partial t} + \vec{v}_j \cdot \nabla \right) \left( \gamma_j \vec{v}_j \right) = \frac{q_j}{m_j} \left( \vec{E} + \frac{1}{c} \vec{v}_j \times \vec{B} \right),
\]

where \( \vec{v}_j \) is the bulk velocity of each fluid, \( \vec{E} \) and \( \vec{B} \) are the electric and magnetic fields, respectively, \( m \) is the particle mass, and \( c \) is the speed of light. \( j = p \) for positrons, and \( j = e \) for electrons, we find that finite amplitude Alfvén waves propagating along a constant background magnetic field obey the dispersion relation [5]

\[
\frac{c^2 k^2}{\omega^2} = 1 - \sum_j \frac{\omega_j^2}{\omega \left( \gamma_j \omega - \Omega_j \right)},
\]

where \( \omega_j = (4\pi n_0 q_j^2/m_j)^{1/2} \) is the plasma frequency, \( n_0 \) is the rest density, and \( \Omega_j \) is the gyrofrequency of species \( j \).

Equation (2) is plotted in Fig. 1 in normalized units, for \( a = \omega_{p}^{a}/\Omega_{p} \) = 1 and \( \eta = 0.1 \) the ratio between the Alfvén wave magnetic field amplitude and the background magnetic field.

![Fig. 1. Dispersion relation (2), \( x = \omega/\Omega_p \) vs. \( y = c k/\Omega_p \), for \( a = 1, \eta = 0.1 \).](image)

There is an electromagnetic and an Alfvén branch. However, unlike the non-relativistic dispersion relation, the electromagnetic branch has no low frequency cutoff and extends down to \( \omega = 0 \); and the Alfvén branch does not exhibit a resonance at the positron gyrofrequency, but is constrained to a shorter frequency range. Also, the Alfvén branch ceases to exist for \( y = c k/\Omega_p > y_c = a/\eta \).

3. Numerical approaches

In order to investigate the consequences of the new features of the dispersion relation for relativistic Alfvén waves, several numerical approaches, in various stages of development, are being implemented.
3.1 Fluid simulation

It is possible to write the fluid equations governing the system in the form:

\[
\frac{d\vec{C}}{dt} = \vec{F}(\vec{C}),
\]

(3)

where \(\vec{C}\) is a multidimensional vector containing the dynamical variables of the problem (velocities, densities, electromagnetic fields), and \(\vec{F}\) is some operator. Equation (3) is then numerically integrated in time, with a rationalized Runge-Kutta algorithm. Preliminary results show that it is possible to perform very stable numerical integrations of the system, for millions of time steps, equivalent to tens of thousands of gyroperiods, if the initial conditions are that of a normal mode of the system.

However, Fig. 1 shows that for a given frequency \(\omega\), the existence of Alfvén waves and the number of wave modes depend on the physical parameters of the plasma. This poses a number of new questions. For instance, we could consider the problem of normal incidence of a plane wave on a density interface. For any Alfvén wave, several modes exist with equal frequency. Moreover, if an Alfvén wave exists with a given frequency in one side of the interface, then if the density is small enough on the other side it could not exist. Does it become an evanescent wave? Is it converted to other modes?

Our next goal is to apply the fluid simulation to these questions, studying the interaction of Alfvén waves with a density discontinuity in the plasma.

3.2 Full particle simulation

Electromagnetic full particle simulations have also been performed. When a moderately large amplitude Alfvén wave propagates in the system, a cascade of parametric decays can be observed [5]. However, for larger amplitudes the energy of an initial normal mode is distributed in low wavenumber modes in a very steep transition (see Fig. 2).

Although further analyses are needed, we intend to use them to study the parametric instabilities for relativistic waves in pair plasmas, extending published results [5].

3.3 Oblique propagation

We are also interested in the issue of oblique relativistic Alfvén waves. This problem cannot be solved analytically, but it is expected that its dispersion characteristics show atypical features analogous to those noticed in Fig. 1. Basically, our approach is to consider the fluid equations in a reference frame moving with the wave phase velocity \(V\). Equations (3) are then written as first-order differential equations for the single variable \(\xi = x - Vt\). Given a particular set of initial conditions, numerical integration is performed, normal modes of the system being given by periodic solutions of the equations.

With this method, several solutions can be found, but their wavenumbers and propagation angles can only be determined a posteriori. A typical result is shown in Fig. 3:

Most solutions found lie along the curve for \(\theta = 0\) [Eq. (2)], which is expected as the seed values satisfy the parallel dispersion relation, but some additional points can be seen. These points should belong to the dispersion relation for some angle \(\theta \neq 0\). Currently, further work is being done in improving this method and others to attack the problem of oblique propagation.

Acknowledgments. One of us (V.M.) wishes to acknowledge a Postdoctoral Fellowship granted by JSPS (Japan).

References


Fig. 2. Particle simulation for a relativistic Alfvén wave propagating in an electron-positron plasma. Wavenumber \(k\) versus time \(t\) is plotted for \(\eta = 1\), pump wavenumber \(k_0 = 1.96\), plasma frequency \(\omega_{pe}/\Omega_p = 10\), and species thermal velocity \(v_{th}/c = 0.02\).

Fig. 3. Numerical investigation for relativistic Alfvén waves propagating obliquely with respect to the background magnetic field. \(\eta = 0.3\).