Three-dimensional magnetohydrodynamic simulation of the sun-solar wind system on the dodecahedronal gridpoint model

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The Solar Terrestrial Environment (STE) is constructed from several regions interacting with each other. Each region has its own time scale, spatial scale and physical parameters quite different from those in the other regions. When we calculate the dynamics of the STE system, it is necessary to make a spheroidal unstructured grid system which covers the whole complex regions. We introduce a triangulate grid system made from dodecahedrons which we call dodecahedronal gridpoint model that satisfies the above requirement. We first calculated the potential field of the sun by using conjugate gradient method on the dodecahedronal gridpoint system. We used the photospheric magnetic field observed by the Wilcox Solar Observatory as a boundary condition. In the MHD simulation (from 1 to 90 solar radii), this potential field that matches the Wilcox observation of the photospheric field is used as an initial condition. The MHD simulation calculates the time development solution of the solar wind using the Finite Volume TVD scheme. In this calculation, we set the plasma outflow at the photosphere as a free parameter. As a result, we obtained a solution that reproduces the features of the solar wind in reality.

1. Introduction

The phenomena that occur in the STE are characterized by complex MHD topology emerging there, and it is convenient to use a spheroidal grid system for such configuration. The dodecahedronal gridpoint model which we introduce here, is one of the most suitable grid systems. From a dodecahedron tangent to a sphere, we can make a triangulate lattice covering a sphere. By piling up many spheres sharing the same center, we can make a 3D grid system which has densely (sparsely) distributed lattice points on the inner (outer) sphere. This grid structure enables fast and accurate calculations for real systems having a source magnetic field inside, such as the STE system in which the magnetic field originate from the centers of the Sun and the planets. In this paper, we apply the grid system to solve the dynamics of the sun-solar wind system.

2. Calculation Method

2.1 Grid System

We will introduce how to make the 3D triangulate grid system made from dodecahedrons. First, we make a regular dodecahedron, which has 12 regular pentagon (Fig.1(a)). The each respect of equilateral pentagon can be divided into five triangles (Fig.1(b)). Next, each triangles can be divided into four triangles (Fig.1(c)). We divide the triangles more and more in this way (Fig.1(d),(e)). As the result, we obtain the grid system which has many lattice points on a spherical surface.

2.2 Initial Condition

If one can observe the intrinsic magnetic field of the sun, we can use it as an initial condition of our system, but it is impossible to observe the field. However, we can obtain the field at the photospheric surface. Thus, for the region outside the photosphere, we calculate a potential field that matches the observation at the photosphere.

If it is assumed that the current doesn’t flow in the region outside chromatospere, the magnetic field \( \mathbf{B} \) there can be described by the scalar potential \( \phi \) as follows:

\[
\mathbf{B} = \nabla \phi,
\]

(1)

So, we get the following Laplace equation:

\[
\nabla \cdot \mathbf{B} = \nabla^2 \phi = 0.
\]

(2)

We solve this Laplace equation using the conjugate gradient method and, as a boundary condition, photospheric magnetic field data observed by the Wilcox Solar Observatory.

2.3 MHD simulation

To compute MHD solutions, we follow Tanaka[1994], and the following 4 main MHD equations (eq.(3)-(6)) are integrated by the Finite Volume TVD scheme in the 3D grid sys-
tem we introduced in section2.1. MHD equations are that
\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0, \tag{3}
\]
\[
\frac{\partial \mathbf{m}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{m} = -\nabla P + \frac{1}{\beta} (\nabla \times \mathbf{B}) \times \mathbf{B} + \rho g - 2\rho \mathbf{\Omega} \times \mathbf{v} - \rho \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}), \tag{4}
\]
\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot \left( \frac{\mathbf{m} \mathbf{B}}{\rho} - \frac{\mathbf{B} \mathbf{m}}{\rho} \right) = 0, \tag{5}
\]
\[
\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \left( \frac{\mathbf{m}}{\rho} (\mathbf{U} + P + \frac{\mathbf{B}^2}{2\beta}) - \frac{\mathbf{B} (\mathbf{m} \cdot \mathbf{B})}{\rho \beta} \right) = 0, \tag{6}
\]
\[
P = (\gamma - 1) \left( U - \frac{\rho u^2}{2} - \frac{\mathbf{B}^2}{2\beta} \right), \tag{7}
\]
where \(\rho\) is the plasma density, \(\mathbf{v}\) is the plasma velocity, \(\mathbf{m}\) is the momentum, \(P\) is the pressure, \(g\) is the gravitational acceleration, \(\mathbf{\Omega}\) is the angular velocity of the sun’s rotation, \(\mathbf{r}\) is the distance from the solar surface. \(U\) is the total energy, \(\gamma = 1.05\) is the ratio of specific heats, and the constant \(\beta\) is the ratio of the static pressure to the magnetic pressure.

### 2.4 Boundary Condition

We set the boundary condition in MHD calculation as shown in Table 1. The term ‘fix’ means that we set a certain value as a boundary condition. The term ‘float’ means that the derivative of the variable is set to zero at the boudary.

<table>
<thead>
<tr>
<th>Inner Boundary</th>
<th>Outer Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>(L)</td>
<td>(1Rs)</td>
</tr>
<tr>
<td>(\rho)</td>
<td>fix</td>
</tr>
<tr>
<td>(v_{para})</td>
<td>float</td>
</tr>
<tr>
<td>(v_{perp1})</td>
<td>fix</td>
</tr>
<tr>
<td>(v_{perp2})</td>
<td>float</td>
</tr>
<tr>
<td>(B_t)</td>
<td>fix</td>
</tr>
<tr>
<td>(B_{t1})</td>
<td>float</td>
</tr>
<tr>
<td>(B_{t2})</td>
<td>float</td>
</tr>
<tr>
<td>(U)</td>
<td>fix</td>
</tr>
<tr>
<td>(P \approx 0)</td>
<td></td>
</tr>
</tbody>
</table>

In Table 1, \(L\) refers to the calculation space, \(Rs\) is the solar radius, \(v_{para}\) is the paraell component of the plasma velocity, \(v_{perp1}\) and \(v_{perp2}\) are the perpendicular components of the velocity, \(B_t\) is the radial component at the solar surface, and \(B_{t1}, B_{t2}\) are the tangential components.

### 3. Simulation Result

We show a typical MHD solution in Fig.2. This solution shows the open magnetic field lines expanding from the polar coronal holes and from the equatorial coronal hole at the leftside of the figure. The figure also shows that the plasma outflows from the narrow gray surface area which is located at the separation of two closed magnetic field lines. We suppose that it is the source of the plasma flow near the active regions.

### 4. Discussion

Fig.3 shows the comparison of the coronal hole predicted by MHD calculation of ours with the one identified in an EIT(Extrem-Ultraviolet Imaging Telescope) image on 00:00 AM, December 9, 2003. The shape of the coronal hole we have obtained looks like the one seen in the EIT image. It proves that our calculation system is correct to some degree in the region where the magnetic pressure is predomiant. However, our simulation result doesn’t reproduce the detailed structure of the solar wind like a high-speed solar wind or a low-speed solar wind. Our simulation result also doesn’t accurate distribution of plasma temperature. Thus, it is necessary to take thermodynamics processes into consideration.

### References
