An new approach to the process of the magnetosphere–ionosphere coupling in global MHD simulations

A. Yoshikawa\textsuperscript{1} and H. Nakata\textsuperscript{2}

\textsuperscript{1}Department of Earth and Planetary Sciences, Kyushu University, Fukuoka, Japan
\textsuperscript{2}Graduate school of Science and Technology, Chiba University, Chiba, Japan

We describe a new process of the magnetosphere-ionosphere coupling for global MHD simulations. In global MHD simulations, field-aligned currents and electric potentials interact each other in the region between the ionosphere and the inner boundary of the magnetosphere. In order to determine these field-aligned currents and the electric potentials self-consistently, we consider the boundary condition as the problem of wave reflection, assuming that the field-aligned currents are associated with the shear Alfvén waves. Separating the perturbed components from the correct solutions, the equation of current continuity of these components is considered. Then we determine the perturbed components generated by the magnetosphere-ionosphere coupling. Since the perturbed components of the electric potentials are regarded as inductive, the new process is applicable to the general problems of the magnetosphere-ionosphere coupling.

1. Introduction

Global MHD simulation is the powerful tool for studying the magnetosphere affected by the solar wind. In the global MHD simulations, the coupling between magnetosphere and ionosphere is also an important factor because the magnetosphere and the ionosphere are connected by field-aligned currents (FACs) that carry energy and momentum. In order to calculate the variations of the magnetosphere correctly, the recent global MHD simulations have managed to include the process of the magnetosphere-ionosphere coupling \cite{e.g.,Tanaka,1995;Raeder et al.,2001;Lyon et al.,2004}. They have adopted the following boundary condition of the inner magnetosphere. First, FACs are determined by magnetic perturbations at the inner boundary of the magnetosphere. Mapping the FACs into the ionosphere along the main magnetic field, the ionospheric electric potentials are calculated from the equation of current continuity:

\[ -\nabla \cdot (\Sigma \nabla \phi) = j_\parallel \sin I, \]

where \( \Sigma \) is ionospheric conductivity tensor, \( \phi \) is the ionospheric electric potential, \( j_\parallel \) is FAC, and \( I \) is the inclination of the main magnetic field. The resultant electric potentials are mapped out to the inner boundary of the magnetosphere. Janhunen \cite{1998} have also proposed a similar boundary condition that the electric potentials determined at the inner boundary are mapped into the ionosphere and then FACs are calculated from the electric potentials using Equation (1). The resultant FACs are mapped out to the inner boundary.

However, solutions of global MHD simulations using these boundary conditions are not always satisfied to Equation (1) because these boundary conditions do not consider the continuities of both potentials and FACs. Here, we describe a new process of the magnetosphere-ionosphere coupling which can be adopted as an inner boundary condition of global MHD simulations.

2. The process of the magnetosphere-ionosphere coupling

In the previous boundary conditions, the electric potential or the FAC is calculated from the other parameter and mapped out to the ionosphere. In this study, we introduce additional components that arise due to the magnetosphere-ionosphere coupling. Correct solutions of the field-aligned currents \( (j_{M|I}) \) and the electric potentials \( (\phi_M) \) in the inner boundary of the magnetosphere are separated as follows:

\[ j_{M|I} = j_{M|I\rightarrow M} + \delta j_{I\rightarrow M}, \]

\[ \phi_M = \phi_{M\rightarrow I} + \delta \phi_{I\rightarrow M}, \]

where \( (j_{M|I\rightarrow M}, \phi_{M\rightarrow I}) \) are the solutions determined by a global MHD simulation without the process of the magnetosphere-ionosphere coupling, and \( (\delta j_{I\rightarrow M}, \delta \phi_{I\rightarrow M}) \) are the additional components generated by the process of the coupling. As the correct solutions satisfy Equation (1), we obtain

\[ -\nabla \cdot (\Sigma \nabla (\phi_{M\rightarrow I} + \delta \phi_{I\rightarrow M})) = (j_{M|I\rightarrow M} + \delta j_{I\rightarrow M}) \sin I. \]

Even if the process of the magnetosphere-ionosphere coupling is not included in the global MHD simulation, the MHD variables express temporal variations. In other words, \( (j_{M|I\rightarrow M}, \phi_{M\rightarrow I}) \) are the summation of steady states and perturbations propagated from the magnetosphere. Thus, \( (j_{M|I\rightarrow M}, \phi_{M\rightarrow I}) \) are also separated as follows:

\[ j_{M\rightarrow I\rightarrow M} = j_{M|I\rightarrow M} + \delta j_{I\rightarrow M}, \]

\[ \phi_{M\rightarrow I} = \phi_{M\rightarrow I\rightarrow M} + \delta \phi_{I\rightarrow M}, \]

where \( (j_{M|I\rightarrow M}, \phi_{M\rightarrow I\rightarrow M}) \) are the components which express the steady states and \( (\delta j_{I\rightarrow M}, \delta \phi_{I\rightarrow M}) \) are the perturbations propagating from the magnetosphere to the inner boundary. These equations show that \( (j_{M|I\rightarrow M}, \phi_{M\rightarrow I\rightarrow M}) \) are ex-
pressed as

\[ j_{\| M} = j_{\| M^{0\text{MHD}}} + \delta j_{\| M^{\rightarrow M}} + \delta j_{\| M^{\rightarrow -M}}. \tag{7} \]

\[ \phi_{M} = \phi_{M^{0\text{MHD}}} + \delta \phi_{M^{\rightarrow M}} + \delta \phi_{M^{\rightarrow -M}}. \tag{8} \]

Assuming that the global MHD simulation includes the process of the magnetosphere-ionosphere coupling correctly, \((\delta j_{\| M^{\rightarrow M}}, \delta \phi_{M^{\rightarrow M}})\) are the correct solutions in the previous step. Thus \((\delta j_{\| M^{\rightarrow M}}, \delta \phi_{M^{\rightarrow M}})\) satisfy Equation (1) and we obtain

\[ -\nabla \cdot (\Sigma \nabla (\delta \phi_{M^{\rightarrow M}} + \delta \phi_{I^{\rightarrow -M}})) = (\delta j_{\| M^{\rightarrow M}} + \delta j_{\| M^{\rightarrow -M}}) \sin I. \tag{9} \]

This equation means that the boundary condition is treated as reflection of waves and that the additional component generated by the coupling process, \((\delta j_{\| M^{\rightarrow M}}, \delta \phi_{I^{\rightarrow -M}})\), are regarded as reflection components of \((\delta j_{\| M^{\rightarrow M}}, \delta \phi_{M^{\rightarrow M}})\) at the ionosphere. It is natural that these components are associated with the shear Alfvén waves. Thus, the relations between the FACs and the electric potentials are shown by

\[ \delta j_{\| M^{\rightarrow M}} = V_{A} \nabla \cdot (\varepsilon_{A} \nabla \delta \phi_{M^{\rightarrow M}}), \tag{10} \]

\[ \delta j_{\| M^{\rightarrow -M}} = -V_{A} \nabla \cdot (\varepsilon_{A} \nabla \delta \phi_{I^{\rightarrow -M}}), \tag{11} \]

where \(V_{A}\) is the Alfvén speed and \(\varepsilon_{A}\) is the permittivity in the MHD medium \((\varepsilon_{A} = 1/\mu_{0}V_{A}^{2})\). If the ionosphere has only the Pedersen conductivity and the Alfvén speed is homogeneous, the classical reflection coefficient of the Alfvén waves is derived from Equations (9)-(11) as

\[ R = \frac{\Sigma_{A} - \Sigma_{P}}{\Sigma_{A} + \Sigma_{P}}, \tag{12} \]

where \(\Sigma_{A}\) is the Alfvén conductance \((\Sigma_{A} = 1/\mu_{0}V_{A})\) [e.g., Scholer, 1970].

As a result, we propose the revised boundary condition at the inner magnetosphere as follows, \(\delta \phi_{I^{\rightarrow -M}}\) is solved from \((\delta j_{\| M^{\rightarrow M}}, \delta \phi_{M^{\rightarrow M}})\) determined at the inner boundary using the equation shown by

\[ -\nabla \cdot (\Sigma \nabla (\phi_{M^{\text{MHD}}} + \delta \phi_{I^{\rightarrow -M}})) = (\delta j_{\| M^{\rightarrow M}} - V_{A} \nabla \cdot (\varepsilon_{A} \nabla \delta \phi_{I^{\rightarrow -M}})) \sin I. \tag{13} \]

Then, replacing \(\phi_{M^{\text{MHD}}}\) by \(\phi_{M} = \phi_{M^{\text{MHD}}} + \delta \phi_{I^{\rightarrow -M}}\) in the inner boundary, the MHD variables in the next step are calculated.

As described before, introducing the additional parameters, we have shown the self-consistent process of the magnetosphere-ionosphere coupling. The previous process is expressed by the parameters used in the present study:

\[ -\nabla \cdot (\Sigma \nabla \delta \phi_{I^{\rightarrow -M}}) = \delta j_{\| M^{\rightarrow -M}} \sin I. \tag{14} \]

Thus, the additional parameters in the present study are \(\delta \phi_{M^{\rightarrow -M}}\) and \(\delta j_{\| M^{\rightarrow -M}}\). It is obvious that the continuities of the FACs and the electric potentials are satisfied by these additional components. Thus this process describes the dynamic closures of the FACs. In addition, the term of \(\delta \phi_{M^{\rightarrow -M}}\) acts as the inductive field and assures the conservation law of energy and momentum [Yoshikawa et al., 2002]. It is desirable that this process is adopted not only by the boundary condition of global MHD simulations but by general problems of the magnetosphere-ionosphere coupling or the other ionospheric models.

Off course, this process should be evaluated by comparing with the results determined by the previous boundary conditions. Now we are preparing for the global MHD simulation with the present condition, modifying the TVD scheme developed by Tanaka [1995]. In near future, we would show the results of the global MHD simulation using the new process presented here.

3. Summary

For the self-consistent boundary condition of the global MHD simulation, we have presented a new process of the magnetosphere-ionosphere coupling, considering the boundary condition as the wave reflection problem. To determine field-aligned currents and the electric potentials self-consistently, we consider the additional components associated with the shear Alfvén waves which propagate between the ionosphere and the inner boundary of the magnetosphere. As the magnetosphere receives the reflection components of these additional components, this new process describes the dynamic closures of the field-aligned currents and the electric potentials. Since these additional components of the electric potentials correspond to the inductive components, the new process is applicable to general problems.

References


