Multi-dimensional Quasineutral Particle/Fluid Simulation Approach for Whistlers

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We have developed a new hybrid electromagnetic simulation scheme HEMPIC for whistlers, which is relativistic and fully nonlinear, works in homogeneous or inhomogeneous situations, is not restricted to a single coherent mode, eliminates both the speed-of-light time scale and the electron plasma oscillation time scale, and concentrates simulation resources on the parts of the electron distribution that make kinetic contributions to wave growth. The current version is 2-D/3-V Cartesian, but the scheme can be implemented in 2-D cylindrical or fully 3-D applications. The plasma is represented as a cold-fluid component plus a set of PIC simulation particles. Since whistlers are slow waves and the plasma is quasineutral (QN), the \textit{full} displacement current (not just the solenoidal part) is neglected in Maxwell’s equations. The simulation particles are pushed in standard PIC fashion, but the cold fluid velocity is obtained from Faraday’s and Ampere’s laws in a form which guarantees QN. In the linear regime, the scheme reproduces the “quasi-longitudinal” dispersion relation, and is accurate for propagation angles up to and beyond the resonance cone. We are using the code to study whistler ducting and long-time nonlinear evolution of whistler instabilities; results will be shown for a variety of situations.

1. Introduction

The instability and nonlinear evolution of whistlers depends crucially on the kinetics of cyclotron-resonant electrons, and therefore simulation techniques must resolve the Vlasov behavior of these electrons. [Nunn 1990, Omura 1991] However standard electromagnetic (EM) PIC simulation techniques are extremely inefficient for whistlers, since: (1) Fluid models are sufficient to treat the non-resonant bulk electrons. (2) Fully EM codes require resolution of the Courant condition time scale for waves traveling at c; this should not be necessary for whistlers since they are slow waves with ω/k<<c. (3) Standard EM codes require resolution of the plasma frequency time scale, which also should not be necessary since usually ω<<Ωe<<ωp, where Ωe is the electron gyrofrequency. (4) In standard EM PIC codes the electrostatic component of E derives from Poisson’s equation, with the charge density ρ(x) determined as the very small difference between n_e(x) and n_i(x); hence enormous numbers of simulation particles are needed to reduce noise. In the past, the Darwin model has been used to eliminate the speed-of-light time scale (issue 2), but Darwin codes are very complicated and somewhat inefficient because of the need to separate the irrotational and solenoidal components of the displacement current, and these issues become even worse if quasineutrality is imposed in standard ways to deal with issues (3) and (4). [Hewett 1994] However, we have found that it is possible in a hybrid code, where the bulk electrons are represented as a cold fluid and the resonant electrons are represented as particles-in-cell (PIC), to recast the equations in a self-consistent and much simpler way that eliminates the speed-of-light time scale, eliminates plasma oscillations, and permits efficient computation with time steps on the Ω_e time scale. In this paper we describe the model, and show a few examples of its use.

2. Simulation Model

Our field equations are
\begin{equation}
\nabla \times \mathbf{B} = 4\pi (\mathbf{J}_e + \mathbf{J}_p) / c, \quad (1)
\end{equation}
\begin{equation}
\nabla \times \mathbf{E} = -\mathbf{B} / c, \quad (2)
\end{equation}
\begin{equation}
\nabla \cdot \mathbf{B} = 0, \quad (3)
\end{equation}
where \(\mathbf{J}_e\) is the cold-fluid electron current and \(\mathbf{J}_p\) is the simulation-particle electron current. The subscripts \(c\) and \(p\) will be used throughout for cold-fluid and simulation-particle contributions to any quantity. Note that the \textit{full} displacement current (not just the solenoidal part) is dropped from (1). It is not necessary to use Poisson’s equation explicitly in the scheme. Using also the cold-fluid momentum equation,
\begin{equation}
m_v \dot{v}_e + m_v v_e \nabla v_e = -e\mathbf{E} - ev_e \times \mathbf{B} / c, \quad (4)
\end{equation}
and the quasineutrality assumption
\begin{equation}
n_e(x,t) + n_p(x,t) = n_i(x), \quad (5)
\end{equation}
where the ion density \(n_i\) is assumed to be time-independent, the equations can be recast in the form
\begin{equation}
\frac{\partial \mathbf{J}_e}{\partial t} = -\frac{\varepsilon}{4\pi} \nabla \times \nabla \times \mathbf{E} - \mathbf{J}_p, \quad (6)
\end{equation}
\begin{equation}
\nabla \times \nabla \times \mathbf{E} + \frac{\varepsilon}{c^2} E - \frac{\varepsilon}{c^2} E - \frac{4\pi}{c} (v_e \mathbf{J}_e) - \frac{\varepsilon}{c^2} v_e \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}_p, \quad (7)
\end{equation}
where $\omega_{pe}^2 = 4\pi n_e e^2/m$. Equation (6) [rather than the momentum equation (4)] is used to advance the cold-fluid electron current $J_c$, and Eq. (7) is used to calculate $E$. Note that Eq. (6) guarantees that $\nabla \cdot (J_e + J_p) = 0$, which insures that if QN is satisfied at one time step, it will continue to be satisfied at the next time step. In the source terms on the RHS of (6,7), $B = B_0 + \delta B$, where $B_0$ is the ambient field and the contribution $\delta B$, which is only significant in the nonlinear regime, is obtained from (1,3). $J_p$ is obtained by pushing the simulation particles (fully relativistically) and laying them down on a grid in the standard manner, and the time derivative $J_p$, which occurs on the RHS of (6,7), is obtained by constructing a stress tensor $K_p(x)$ from the particle positions and momenta at any given time, and using the warm-fluid momentum equation for the particles,

$$J_p = \frac{c}{m} \nabla \cdot K_p + \frac{n e^2}{m} E + \frac{4\pi n_e e^2}{mc} v_p \times B. \quad (8)$$

When (8) is substituted, Eq. (7) takes the more revealing form

$$\nabla \times \nabla \times E + \frac{\omega_p^2}{c^2} E = \frac{4\pi e}{c^2} \nabla \cdot (n_e v_e v_e + K_p) - \frac{4\pi e B \times (J_e + J_p)}{mc}. \quad (9)$$

It is important to note that quasineutrality, which is built into our scheme, does not mean that there are no electrostatic fields, only that $|n_i - n_e| \ll n_i$, and that the electrostatic component of the electric field $E$ is determined by the requirement to maintain QN, rather than by solving Poisson’s equation.

3. Applications

We have begun to apply HEMPIC to nonlinear studies of whistler instabilities, with the goal of addressing multidimensional evolution in inhomogeneous environments. For illustrative purposes, a convenient 1D/3-V test problem, amenable to analytic solution, is the whistler instability with $k$ parallel to $B_0$, for an electron distribution of the form

$$f(p) = n_e \delta(p) + \frac{n_p}{2\pi p_{\perp,0}} \delta(p_{\parallel} - p_{\parallel,0}) \delta(p_z - p_{z,0}), \quad (10)$$

i.e. a cold plasma component plus a ring distribution of momenta $p$. In the linear regime, the code reproduces calculated dispersion relations and growth rates to within accuracy of a few percent, with time steps of the order of $1/8\Omega_e$. The nonlinear evolution is also faithfully represented. Figure 1 shows a snapshot of the simulation particle positions in three different sections of phase space: (a) $\psi$ vs. $\alpha$, where $\psi$ is the difference in phase between the wave magnetic field and the particle transverse velocity, and $\alpha$ is the pitch angle; (b) $v_x$ vs. $z$; and (c) particle velocity phase $\phi$ vs. $z$. (b) and (c) clearly show the phase bunching concommitant with wave growth, while (a) shows the phase trapping that leads to saturation. [Helliwell 1973]

A second area of code application is whistler ducting. We have recently developed a full-wave theory of ducting (not yet published), which (unlike geometrical optics theories) is applicable to narrow and sharp-edged density channels. Concommitant HEMPIC studies show good agreement with the analytic theory. Figure 2 shows a 2-D HEMPIC simulation of ducting in a reduced-density channel.

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**References**