Nonlinear frequency shifts of the dust ion acoustic wave and the dust acoustic wave

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The nonlinear frequency shifts of a low-frequency, coherent dust ion acoustic wave and a dust acoustic wave in the presence of turbulence are investigated in the framework of weak turbulence theory. It is found that the frequency shifts of the dust ion acoustic wave and the dust acoustic wave in an unmagnetized dusty plasma are always positive irrespective of the propagation direction of the coherent wave, as in the case of the customary ion acoustic wave in the absence of dust particles. The magnitudes of the frequency shifts of dust ion acoustic wave and dust acoustic wave have shown overall decreasing behavior with increasing dust density, but the nonlinear frequency shift of a dust acoustic wave is found to be more pronounced than the case of the dust ion acoustic waves.

1. INTRODUCTION

Recently, the problem of nonlinear frequency shift has been reformulated within the context of the weak turbulence theory [1]. We extend this formalism to study the nonlinear frequency shift of the dust ion acoustic and the dust acoustic wave. The problem of the nonlinear frequency shift of the usual ion acoustic wave has been investigated long time ago [2, 3]. The dust ion acoustic and the dust acoustic wave behave in a manner very similar to the customary ion-acoustic wave in a fully ionized plasma. Thus, one may naively expect that a similar behavior of frequency shift would appear for the dust ion acoustic wave (DIAW) and the dust acoustic wave (DAW). We find that certain features agree with this expectation but a new feature emerge, especially for the dust acoustic wave: The nonlinear frequency shift of a dust ion acoustic wave (DIAW) is qualitatively similar to the customary results involving ion acoustic wave (IAW) in fully ionized plasmas. The magnitude of the frequency shifts of dust ion acoustic wave (DIAW) and dust acoustic wave (DAW) decrease with increasing dust density [4]. However, the nonlinear frequency shift of a dust acoustic wave (DAW) is found to be more pronounced than in the case of the customary ion acoustic waves [5].

2. NONLINEAR FREQUENCY SHIFT OF LINEAR DUST ION ACOUSTIC WAVES by DUST ION ACOUSTIC TURBULENCE

We consider an unmagnetized static dusty plasma. It is assumed that the dust grains are negatively charged, and that the charge does not change during the dynamical processes. We thus consider the number of electric charges associated with the dust ions to be fixed. We obtain the desired dispersion relation for the coherent DIAW, corrected by the presence of background DIAW turbulence [4], namely,

$$\omega = \omega_{k}^{DIAW} \left(1 + \sum_{\sigma' = \pm 1} \int d\mathbf{k}' a_{k,k'}^{\sigma DIAW} W_{k'}^\sigma W_{k'}^{DIAW} \right),$$

where $$\omega_{k}^{DIAW}$$ is frequency of the linear dust ion acoustic wave. And $$a_{k,k'}^{\sigma DIAW}$$ is the form factor and $$W_{k'}^\sigma$$ is the normalized turbulent wave energy density defined by

$$W_{k'}^\sigma = \left( \frac{\omega_{\pi i}}{\omega_{k}^{DIAW}} \right)^2 \frac{4\pi n_{e} T_{e}}{\sigma' DIAW}.$$  \hspace{1cm} (2)

If the DIAW turbulence is isotropic, we arrive at the following expression for the angle averaged form factor.

![Fig. 1. Form factor of the frequency shift of DIAW as a function of k'\lambda_De for the case where (a) the coherent and turbulent waves propagate with positive phase velocities along the z axis (\sigma' = +1), and (b) the z components of the phase velocities of the coherent and turbulent waves are opposite (\sigma' = -1). The physical parameters are r = n_{0}/n_{e} = 1.00 for solid line, r = 1.03 for dotted line, r = 1.10 for dashdot line, and r = 1.50 for dashed line.](image-url)

Figure 1 shows the plots of the form factor defined as a function of dimensionless wave number k'\lambda_De associated...
with the turbulent wave. Form factors depend on $r$, the ion to electron density ratio, which in turn depends on the density of dust particles by virtue of the neutrality condition.

Note that the form factors of frequency shift remain positive irrespective of the sign of $\sigma'$, and that the phase velocity of the coherent DIAW increases due to the interaction with the turbulent DIAWs. The overall characteristics are similar to the frequency shift of the ion acoustic wave caused by interaction with the ion acoustic turbulence [2, 3]. Solid line in Figure 1 is the case of $r = 1$, meaning that there are no dust particles. This is the customary plasma composed of ions and electrons, and thus this case can be directly compared to Figs. 3(a) and 3(b) of Ref. [3]. Two results are similar as indeed they should be. Other lines on Figure 1 show that the magnitude of form factor decreases as density of dust particles increases, i.e., for increasing $r$. From these results, we conclude that the presence of dust particles leads to the suppression of frequency upshift of DIAW.

3. NONLINEAR FREQUENCY SHIFT OF LINEAR DUST ACOUSTIC WAVES

We obtain the dispersion relation for the dust acoustic wave, corrected by the presence of background dust acoustic turbulence [5], namely,

$$\omega = \omega^{DAW}_{k} \left( 1 + \sum_{\sigma' = \pm 1} \int d\mathbf{k}' \sigma'^{DAW}_{k,\mathbf{k}'} W^{\sigma'^{DAW}}_{k} \right),$$  

(3)

where $\omega^{DAW}_{k}$ is frequency of the linear dust acoustic wave. $\sigma'^{DAW}_{k,\mathbf{k}'}$ is a form factor and $W^{\sigma'^{DAW}}_{k}$ is the turbulent wave energy density defined by

$$W^{\sigma'^{DAW}}_{k} = \left( \frac{\omega^{pd}_{k} / \omega^{DAW}_{k}}{4\pi n_{e0}T_{e}} \right) I^{\sigma'^{DAW}}_{k,\mathbf{k}'}.$$  

(4)

Figures 2 show angle-averaged form factor $(\sigma'^{DAW}_{k,\mathbf{k}'})$ as a function of the magnitude of dimensionless wave number $k'\lambda_{De}$ associated with the turbulent waves. Note that the form factor $(\sigma'^{DAW}_{k,\mathbf{k}'})$ remains positive irrespective of the sign of $\sigma'$, and that the phase velocity of the coherent wave increases due to the interaction with the turbulent waves. The frequency shift coming from the interaction with the forward turbulent waves is more significant than that from the backward turbulent waves.

Upon comparison with Figs. 3a and 3b of Ref. [3], one may notice that the overall characteristics are qualitatively similar in the both cases (note that we use a logarithmic vertical scale in contrast to Ref. [3] which adopts linear scale). This comparison confirms that the characteristics of the nonlinear frequency shift associated with the dust acoustic mode are qualitatively similar to those of the customary ion acoustic wave [2, 3]. However, a distinguishing feature of the frequency shift of dust-acoustic waves is, as demonstrated in Figs. 2, that the magnitude of the frequency shift is much greater than that of the usual ion acoustic wave. Upon comparison with Ref. [3], one may note that the present result gives a frequency shift form factor which is higher by many orders of magnitude. This implies that the nonlinear frequency shift of the dust acoustic wave is much more sensitive to the presence of a turbulent background.

4. CONCLUSION

We have studied the effects of dust particles on the nonlinear frequency shifts of DAW and DIAW. Although the linear dispersion relations of dust acoustic and dust ion acoustic wave look similar to that of the ion acoustic wave in a customary plasma, the nonlinear frequency shifts of these waves in a dusty plasma are distinctly different from each other, when compared to that of the ion acoustic wave.

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References