Equilibria and Dynamics of Dust in Flowing Plasma

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We discuss a framework for performing both theoretical analyses and simulations of dusty plasma, in which the dust grains interact with each other via a dynamically-shielded potential \( \Phi(r) \), and plasma electrons and ions appear only through a dielectric function that is embedded in \( \Phi(r) \). If the plasma ions are flowing, \( \Phi(r) \) is asymmetric upstream and downstream, which has very remarkable consequences. We show that self-bound flow-aligned “dust molecules” can form, and that these molecules propel themselves upstream against the flow. For the case of two grains confined within an axisymmetric external potential well, we show that the grains move through a hysteresis cycle, including discontinuous jumps, as the external parameters are slowly varied. If the well is anharmonic, the hysteresis cycle includes oblique (symmetry-breaking) states. We also show several cases where grains can go into large-amplitude self-excited oscillations which persist in spite of friction. Many of these phenomena have been observed, and we shall connect the theory and simulations to experiment.

1. Introduction

In dusty plasma physics, the focus of interest is usually the dust grains rather than the ions and electrons. The grains acquire a negative charge \(-Ze\) with \( Z > 1000 \). Often, a dust cloud constitutes a strongly coupled plasma component, i.e. the typical grain-grain interaction energy greatly exceeds the average grain kinetic energy. This leads to condensation of the dust into crystalline structures. Investigators can follow the details of structure and of dynamical processes such as phase transitions, at a level of detail that is impossible in any other system. However, it is found that the structures and transitions in dusty plasma are not the same as those found in ordinary solids and fluids. For example, in a dusty plasma where the ions are flowing, the crystal structure most often seen is the simple hexagonal crystal, in which the grains line up into crystal planes transverse to the direction of ion flow, with hexagonal structure within each plane, but with grains stacked one above the other to form columns aligned with the flow. This very anisotropic structure is never seen in ordinary matter, and it is a result of the unique features of the grain-grain interaction in dusty plasma. These features also result in remarkable structural and dynamic properties even in a system with only a few dust grains. We shall discuss an approach that can be used as the basis for analytic theory, and also for simulation codes, and we shall apply these techniques to few-grain and low-dimensional configurations in dusty plasma.

2. Grain-grain interaction

In dusty plasma, the dust grains function on time scales many orders of magnitude slower than the plasma ions and electrons. Therefore it is impractical to represent the plasma in a simulation model on the same basis as the dust. Typically the dust grains may number anywhere from one to thousands, whereas the ions and electrons may number \( 10^{12} \) or more, and one wants to treat the grains as discrete particles but to represent the plasma as a continuous medium. This can be done in a very simple way.

The charged dust grains in a plasma interact via a dynamically shielded potential \( \Phi(r) \), which can be calculated in the standard way from linear response theory [Rostoker 1960]. This model assumes that the ions and electrons are weakly coupled among themselves and to the dust grains (even though the dust grains may be strongly coupled among themselves). This is a reasonable approximation quite generally, since the ion-dust interaction scales as \( Ze^2 \), whereas the grain-grain interaction scales as \( Z^2 e^2 \), and it is a very good quantitative approximation for the common case where ions are flowing with a stream velocity \( u_i \) of order \( c_s \), because both the ions and electrons then have a large kinetic energy of order \( T_e \). The shielded potential \( \Phi(r) \) depends on the parameters \( u_i/c_s, n_i T_e, n_i T_e, \) and \( v/\omega_{pi} \), where \( c_s \) is the ion sound speed, \( v_i \) is the ion collision frequency, and \( \omega_{pi} \) is the ion plasma frequency, and for any given set of parameters \( \Phi(r) \) can easily be calculated numerically. One can then use \( \Phi(r) \) as the grain-grain interaction in a particle simulation code that includes only the grains as simulation particles. The plasma is then completely eliminated from the problem; it appears only implicitly within \( \Phi(r) \). We have used this model as the basis for analytic theories, and also for a simulation code DSD (Dynamically Shielded Dust). In various applications, we use either PIC or molecular dynamics (pair interaction) approaches in DSD, or in some cases we use a combination of both techniques [Joyce 2001; see also Ganguli, invited talk in these Proceedings].

If there is no plasma flow, \( \Phi(r) \) reduces to simply the Debye-shielded Coulomb potential, which was the starting point for dusty plasma theory twenty years ago. But if the ions are flowing, as is usually the case in both laboratory discharges and space plasmas, \( \Phi(r) \) has a very different nature, which is shown in Fig. 1. Upstream \((z>0)\) and to the side, \( \Phi(r) \) represents a short-range repulsive potential (similar to the Debye potential), but downstream \((z<0)\) it constitutes an oscillatory wakefield. The asymmetry upstream and downstream makes this potential very different from the usual isotropic interaction between particles. The downstream wakefield includes nodes which attract other dust grains, i.e. for certain particle separations, a negatively-charged grain can exert an attractive force on other negatively-
charged grains, but only on downstream grains. Consequently, Newton’s third law does not hold and neither energy nor momentum is conserved. This leads to remarkable dynamical consequences, which we shall explore in this paper, using both analytic and simulation techniques. Some of this material has recently been published [Lampe 2005].

3. Dust molecules

Consider two grains that are aligned along the ion flow direction $z$ (vertically), with no other force acting on the grains except the interaction force $F = -\nabla \Phi$ between the two grains and frictional drag against the neutral gas. The vertical component $F_z$ of $F$ is shown in Fig. 2. Notice that $F_z$ is repulsive upstream ($z>0$), but that $F_z$ is attractive downstream for $-4 < z/\lambda_{De} < -0.5$. Thus, if the grain separation is $z_{12}$ and it happens that $F_z(z_{12}) = F_z(-z_{12})$, a steady state will occur in which the upstream grain pulls the downstream grain with force $F_z(z_{12})$, while the downstream grain pushes the upstream grain with force $F_z(-z_{12})$. For the parameters of Fig. 2, this occurs at $z_{12} \approx 1.3 \lambda_{De}$. Furthermore, for the case of two grains we can define an effective energy and show that this configuration corresponds to a minimum of an effective potential, and thus is stable to both longitudinal and transverse perturbations. This is thus a self-bound dust molecule, made up of two negatively-charged grains that one might expect to repel each other. This molecule does not just sit stationary in the gas; it propels itself upstream against the ion flow, since each grain exerts an upstream force on the other grain. Eventually the molecule reaches a steady velocity, determined by the balance of this force against neutral drag.

We show that this argument can be extended to predict bound molecules consisting of three or four flow-aligned grains. However, there is no constant of the motion (effective energy) if there are more than two grains, and three- and four-grain molecules are subject to a transverse vibrational instability. Increasing the neutral pressure $P$ has a strong stabilizing effect, as the last grain downstream is not in equilibrium and always escapes. We shall show simulations of the self-assembly of dust molecules, self-excited oscillations, and the temporary formation and break-up of longer strings.

4. Two Grains in an External Confining Potential

In experiments, the dust grains normally levitate in the sheath above the lower electrode, and are subject to a strong vertical confining force, which is the resultant of gravity (constant), ion drag (constant), and the sheath electric field (rapidly decreasing with $z$). In addition, the electrode is normally configured to provide a weak confining potential in the horizontal plane. Thus, we may assume that the dust grains are subject to an anisotropic, axially symmetric potential $V(r,z)$, in addition to the grain-grain forces. Similar circumstances can occur in space plasmas. We now consider the case of two grains in such a potential well.

For two grains in a harmonic potential, we show that an effective energy can be defined, such that stable equilibria correspond to minima of an effective potential. All such equilibria have the two grains either vertically aligned (favored when the vertical external confining force $F_{ext} = \nabla V/\partial z$ is weak) or located symmetrically in the mid-plane (favored when $F_{ext}$ is strong). In an intermediate range, both types of equilibria exist simultaneously. As $F_{ext}$ is slowly varied, the grains move through a hysteresis loop, which involves discontinuous jumps from one type of equilibrium to the other.

In the general case of two grains in an axisymmetric anharmonic potential, there is no constant of the motion. In this case axisymmetry is broken by stable oblique equilibria in which one of the two identical grains moves below the mid-plane, the other sits above the mid-plane, and neither is on axis. We solve analytically in the case of quadratic/quartic potentials, finding that as the strength of vertical confinement is varied, the grains trace out a hysteresis loop including oblique alignments and discontinuous jumps. The analytic theory and simulations for this case show good agreement with the experiments of [Steinberg 2001]. Furthermore, we find a parameter range where an equilibrium exists with both grains sitting symmetrically in the mid-plane, but the equilibrium is unstable to vertical oscillations. In this case, the grains go to a limit cycle of self-excited large-amplitude oscillations, which persist indefinitely despite friction. This phenomenon has also been seen in experiments [Nunomura 1999, Samarian 2001, Takamura 2001, Ticos 2004].

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References