Non-linear Ion-Acoustic Resistivity: Ensemble Vlasov Simulations

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Magnetic reconnection in collisionless plasmas requires the violation of ideal MHD by various kinetic-scale effects whose relative importance is uncertain. Recent research has highlighted the potential importance of wave-particle interactions by showing that Vlasov simulations of unstable ion-acoustic waves predict an anomalous resistivity that can be higher than a popular analytical quasi-linear estimate. We investigated the non-linear evolution of the ion-acoustic instability and its resulting anomalous resistivity by examining the properties of a statistical ensemble of 104 Vlasov simulations. The simulations differ in their initial electric noise field but are otherwise identical. The instability is produced in a Maxwellian electron-ion plasma with electron to ion temperature ratio $T_e/T_i = 2$, $T_e = 2$ eV, representative of the earth’s magnetopause. We study the evolution of the ensemble probability distribution of anomalous resistivity values produced during the linear, quasi-linear and non-linear evolution of the instability. We show that the ensemble mean of the ion-acoustic resistivity during the non-linear regime is higher than estimates at quasi-linear saturation and that the ensemble standard deviation is comparable to the ensemble mean, providing a natural source of localized resistivity for fast reconnection.

1. Introduction

In collisionless space plasmas, magnetic reconnection requires the breakdown of the ion and electron frozen-in condition at the magnetic null point. The role of wave-particle interactions, as a mechanism to decouple electrons from the field, remains an open question [e.g. Drake et al., 2003; Omura et al., 2003; Petkaki et al., 2003; Silin and Büchner, 2003]. Some wave modes like lower-hybrid waves and whistler mode waves require the presence of a magnetic field, but ion-acoustic and Buneman waves can exist in the heart of the current sheet, where the magnetic field becomes small [Drake et al., 2003; Hellinger et al., 2004].

Anomalous resistivity can be provided by electron momentum changes through ion-acoustic wave-particle interactions [e.g. Coroniti and Eviatar, 1977; Galeev and Sagdeev, 1984]. Recent simulation work by Watt et al. [2002] has shown that the analytical calculations of ion-acoustic anomalous resistivity by Labelle and Treumann [1988] have underestimated its level, possibly because the details in the way the particle distribution functions (DF) are changing are not taken into account [Watt, 2001; Watt et al., 2002; Petkaki et al., 2003].

The anomalous resistivity continues to grow after quasi-linear saturation. In order to quantify the behavior of the ion-acoustic anomalous resistivity at and beyond quasi-linear saturation, we must use a statistical ensemble approach. The anomalous resistivity behavior in this nonlinear regime is sensitive to the initial noise electric field conditions of the simulation. We performed 104 Vlasov Simulations with identical initial conditions except for the initial white noise electric field.

2. Model and Results

We study the evolution of unstable ion-acoustic waves and associated anomalous resistivity using a finite difference approximation to the Vlasov-Maxwell equations. We use a one-dimensional (1D) electrostatic Vlasov simulation code with periodic boundary conditions [Watt, 2001]. Electrons and protons are modeled using Maxwellian DFs. For electrostatic waves, the only force acting on the plasma is that due to an electric field, it is assumed that the magnetic field is zero. The electric field is integrated forward in time using Ampère’s law. We assume that the external current balances the spatially-averaged internal current. The Vlasov-Maxwell equations are integrated forward in time using the MacCormack method. The resistivity $\eta$ is calculated at each timestep in the simulation using the rate of change of electron momentum [Davidson and Gladd, 1975].

The dimensions and resolution of the ion and electron phase space grids are calculated by the simulation program to suit the individual set of initial parameters [Watt, 2001]. A full description of how we set up the simulations is presented in Petkaki et al. [2003]. In the simulations presented here the number of spatial grid points is $N_s = 642$. The number of velocity grid points for electrons is $N_{v,e} = 891$ and for ions is $N_{v,i} = 289$. The Vlasov simulation runs were performed for a temperature ratio $T_e/T_i = 2$, ion temperature $T_i = 1$ eV, number density $n = n_i = n_e = 7 \times 10^6 \text{ m}^{-3}$, representative of the earth’s magnetopause. The mass ratio used is $m_i/m_e = 25$, but some simulations were repeated for higher mass ratios. Simulations with higher mass ratios confirm qualitatively the behavior seen in the evolution of the ion-acoustic anomalous resistivity for $m_i/m_e = 25$.

We performed Vlasov simulations for 104 different initial white noise electric fields applied at $t = 0$. For each of the 104 Vlasov simulations the phases of the initial white noise field were randomly picked out of the uniform distribution, otherwise the simulations are identical. Figure 1 shows the whole ensemble evolution of the ion-acoustic anomalous resistivity. In Figure 1a we have overplotted the time evolution of all 104 anomalous resistivities, each one represented by a different color. This representation shows the similarity and diversity in the behavior of the resistivity. In Figure 1b we plot the mean value of the resistivity (continuous black line)
calculated by averaging the value of the 104 resistivity values at each timestep. On the same graph plus/minus three standard deviations from the mean are plotted (continuous red lines). Comparing Figures 1a and 1b we see that the range of resistivity values is well confined in \( \pm 3\sigma \) of the mean, as would be expected by a Gaussian distribution.

3. Discussion and Conclusions

In previous published papers [Watt et al., 2002; Petkaki et al., 2003], the ion-acoustic anomalous resistivity was measured at quasi-linear saturation, defined by the saturation of the fastest growing wave mode. This is estimated to be at about \( \omega_{pe}t = 220 \) in the simulations presented here. The anomalous resistivity continues to evolve after quasi-linear saturation and it is highly variable in the non-linear regime of the ion-acoustic instability in which a natural plasma may be expected to exist. At quasi-linear saturation \( \omega_{pe}t = 220 \), the ensemble mean value of resistivity is \( \eta(t) = 74 \Omega \text{ m} \) and is consequently an underestimation of the anomalous resistivity since the resistivity continues to grow. For example, measurement of the ensemble mean value of the ion-acoustic resistivity maximum at time \( \omega_{pe}t = 253 \) gives \( \eta(t) = 188.7 \Omega \text{ m} \). This value is more representative of the anomalous resistivity in the non-linear regime.

The standard deviation of resistivity continues to grow monotonically (with minor fluctuations) throughout the whole evolution (Figure 1b). At quasi-linear saturation \( \omega_{pe}t = 220 \), the standard deviation \( \sigma = 12 \Omega \text{ m} \), which is 16% of the mean. At saturation of mean value of the resistivity at time \( \omega_{pe}t = 253 \), the standard deviation \( \sigma = 38.7 \Omega \text{ m} \), which is 20% of the mean. By the end of our simulation at time \( \omega_{pe}t = 350 \), the mean value of resistivity is 198 \( \Omega \text{ m} \) and the standard deviation \( \sigma = 105 \Omega \text{ m} \), which is 53% of the mean value of resistivity. This increasing variance is likely attributable to the increasing total wave energy and the increasing number of non-linearly interacting wave modes as the power-law electric field spectrum develops. When we compared the evolution of total wave energy and the standard deviation of the anomalous resistivity, we saw that the two quantities grow similarly in the quasi-linear and non-linear phases.

A natural plasma that supports the ion-acoustic instability may be expected to exist predominantly in the non-linear regime of the instability. Our calculations show that the ensemble mean of the ion-acoustic resistivity during the non-linear regime is higher than estimates at quasi-linear saturation from both analytic [Labelle and Treumann, 1988] and simulation results [Watt et al., 2002; Petkaki et al., 2003], and that the ensemble standard deviation is comparable to the ensemble mean, providing a natural source of localized resistivity for fast reconnection.

References
