Self-organization of parallel propagating Alfven wave turbulence

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Nonlinear evolution of Alfven wave turbulence is discussed within the context of the derivative nonlinear Schroedinger equation (DNLS), a subset of the hall-MHD equation set, which includes quasi-parallel propagating right- and left-hand polarized Alfven wave modes. Via numerical time integration of the equation under periodic boundary conditions, we discuss self-organization of solitary waves, evolution of tri-coherence, synchronization of the wave phases, among others.

Finite amplitude magnetohydrodynamics (MHD) waves evolve nonlinearly via wave-particle and wave-wave interactions. Nonlinear evolution of the waves has been an attracting subject for nonlinear wave research. Also, these waves are considered as an essential ingredient for Fermi acceleration of cosmic rays. Among various nonlinear processes triggered by the presence of the finite amplitude Alfven waves, we concentrate in this presentation on nonlinear evolution and self-organization of Alfven waves caused by self-coupling among Alfven waves which are parallel and uni-directional. By using this particularity, the hall-MHD equation set can be reduced using the method of averaging to yield the so-called DNLS equation,

$$\frac{\partial b}{\partial t} + a \frac{\partial}{\partial x} (|b|^2 b) + i \mu \frac{\partial^2 b}{\partial x^2} = 0$$

$$b = b_x + ib_y, \quad a = \frac{1}{4} \frac{C_i^2}{C_A^2 - C_s^2}, \quad \mu = \pm \frac{1}{2}$$

where $b$ is the complex transverse magnetic field normalized to the static field in the $x$ direction, $C_A$ is the Alfven speed, $C_s$ is the sound speed, and time and space are normalized using the ion gyrofrequency and $C_A$. The DNLS is integrable under various boundary conditions. In the present study, we discuss nonlinear evolution of Alfven waves in (1), with particular attention to correlation of wave phases.

In Fig.1 we show an example of time integration of (1), where $|b|$ (envelope) is plotted in the phase space of time ($t$) and space ($x$). Evolution of the power spectrum of the same run is given in Fig.2, in which the vertical axis is the wave number $k$, with $k$ positive (negative) corresponds to right- (left-) hand polarized waves. Initial conditions are given as a superposition of finite amplitude monochromatic waves (the parent) and a very small amplitude white noise. Boundary conditions are periodic.

Until $t \approx 200$, the parent wave energy is gradually transferred to daughter waves through modulational instability, which can be seen as a gradual spreading of the power spectrum in the $k$-space(Fig.2). Around $t \approx 300$, a series of solitary waves are born (Fig.1), corresponding to broadening of the spectrum in Fig.2. These solitary waves repeatedly appear and vanish, while interacting with each other. It is also noted that the almost all the solitary waves disappear at certain time intervals like $t \approx 900$, which is indicative of the presence of (near) recursion of the DNLS system with periodic boundary conditions.

The nonlinearity of the DNLS is cubic, i.e., the four wave resonance is the basic nonlinear interaction,

$$k_1 = -k_2 + k_3 + k_4$$

where similar relation is held for frequencies as well. Since it turned out that the power of the parent wave ($k0=-11$) remains to be the most dominant one throughout the run, we evaluate the tri-coherence among the wave modes with $k3=k4=k0$ above. Then it is now clear that the modulational instability generates two side band waves around the parent wave, since we can write $k1=k0-kd, k2=k0+kd$, with arbitrary $kd$. Using these specifications of the wave numbers, we plot time evolution of sine of the relative phase among the four waves in Fig.3. As we will discuss in the presentation in detail, this parameter gives indication of energy (quanta) flow among the wave modes, which explains generation and decay of the solitary wave rows observed in the simulation.

In the presentation, we also discuss wave turbulence in the driven-damped DNLS system. We show results of numerical runs with various parameters.

![Fig 1 Time evolution of envelope, $|b|$](image)
Fig 2 Time evolution of power spectrum.

Fig 3 Time evolution of sine of the relative phase ($\theta_k$) among the four waves satisfying $k_1+k_2=2k_0$.

Fig 4 Relation between the “flow of quanta” among the resonant wave modes (horizontal axis) and the rate of which the relative phase among the resonant wave modes varies (vertical axis). The flow of quanta among the sites is enhanced (suppressed) when the relative phase is almost constant (varies rapidly) in time.

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References