Numerical Simulation of a Self-Field MPD Thruster using Lax-Friedrich Scheme

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Numerical modeling of the self-field magnetoplasmadynamic (MPD) thruster is performed. The second order TVD Lax-Friedrich scheme is used in order to solve the time-dependent resistive MHD equations with nonequilibrium ionization. Although a simplified model without viscosity and thermal conduction is adopted, the model reveals the flow structure of so-called cathode jet, which features both high-temperature and high-density plasma jet emanating from the cathode tip as a result of strong Joule heating.

1. Introduction

NASA announced in January 2003, to develop a nuclear power generating system that could ultimately be used for a manned mission to Mars. High-power electric thrusters suitable for such a mission are intensively studied in the U.S., in ESA countries, and in Japan. A self-field magnetoplasmadynamic (MPD) thruster is one of the electric propulsion devices.

The feature of the self-field MPD thruster is the acceleration of the plasma by the Lorentz force by the discharge current and the induced magnetic field [Fig. 1]. Since both ionization and acceleration occur in the same chamber, the complicated physical phenomenon in the thruster makes it difficult to obtain the comprehensive information to optimize the performance, hence our goal is to accurately model and simulate the processes in the MPD thruster, then obtain some design guidelines for high-power interplanetary missions.

2. Numerical Model

2.1 Numerical method

Characteristic of the self-field MPD thruster is to use the induced magnetic field in order to gain the impulse. Thus, a induction equation of the magnetic field has to be solved to study the unsteady flow of the plasma. Since the induced equation is hyperbolic, upwind or symmetric TVD schemes can be used. However, the extension of these schemes to MHD equation is not a simple task. In this study, to avoid complicated schemes, the second order accurate in time and in space high-resolution Lax-Friedrich scheme is used [2]. This method does not require the calculation of eigenvector matrices which often appears in the general TVD schemes, thus it is useful for solving the MHD equations even if additional effects such as ionization is included in the model.

2.2 Governing Equations

The following MHD equations are used to analyze the thrust generation mechanism: The propellant gas is partially singly-ionized argon. The flow is axisymmetric, electrically neutral, and in thermal equilibrium. The viscosity and thermal conduction, and the Hall effect are not taken into consideration. Then the governing equations used here are as follows.

Mass conservation:
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \] (1)

Momentum conservation:
\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[ \rho \mathbf{u} \mathbf{u} + \left( p + \frac{B^2}{2\mu_0} \right) \mathbf{I} - \frac{BB}{\mu_0} \right] = -\frac{1}{\mu_0} B \nabla \cdot B \] (2)

Energy conservation:
\[ \frac{\partial E_m}{\partial t} + \nabla \cdot \left[ E_m \mathbf{u} + \left( p + \frac{B^2}{2\mu_0} \right) \mathbf{u} - \frac{1}{\mu_0} (\mathbf{u} \cdot B) B \right] + \frac{1}{\sigma \mu_0} j \times B = -\frac{1}{\mu_0} (\mathbf{u} \cdot B) \nabla \cdot B \] (3)

Equation of magnetic induction:
\[ \frac{\partial B}{\partial t} - \nabla \times (\mathbf{u} \times B) + \nabla \times \left( \frac{1}{\sigma \mu_0} \nabla \times B \right) = 0 \] (4)
Mass conservation of the ion:
\[
\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i u) = \dot{\rho}_i
\]  
(5)

where \(E_m\) in (3) represents the total energy including the energy of the magnetic field. The electrical conductivity is given by the Spitzer-Ha"arm formulation [3]. The right hand side of (5) is the ionization rate of argon [4]. In addition, the Maxwell equations, the equation of state and the generalized Ohm’s law are used.

Indeed, the azimuthal magnetic field \(B\) in the induction equation of the magnetic field is converted into \(\psi = rB\) in the calculation.

2.3 Boundary Condition

The temperature, mass flow, ionization degree are fixed at the inlet and, from these value, the pressure, density, and velocity are determined at the inlet using Saha’s equation and the equation of state. The total discharge current has to also given to initial condition.

The outflow boundary is considered as a free stream boundary. For the solid body and symmetry axis, no flow condition through the boundary is taken.

Electromagnetic boundary conditions are specified as follows. At the inflow boundary \(\psi\) is set to \(-\mu_0 J/2\pi\) where \(J\) is the total discharge current. On the electrodes surfaces, the tangential electric field component is fixed as zero. At other boundaries, i.e., at the outlet and symmetric axis, the magnetic flux is set to zero.

3. Results

Calculation is conducted in the case of \(J=1000\text{A},\) mass flow=1.25 g, \(T_{\text{inlet}}=7000\text{K},\) \(\alpha_{\text{inlet}} = 5.0 \times 10^{-4},\) where \(\alpha\) is the ionization degree. Mach number at the inlet is 1.6 under this inlet condition.

Calculated pressure, temperature, and ionization degree distributions are plotted in Figs.2,3, and 4. The plasma pressure rises at the cathode tip due to the Lorentz force toward the symmetric axis, then the cathode tip is pushed by high pressure. This force is called “pumping force”. On the other hand, axial Lorentz force acts on plasma mainly in the interelectrode region because the most current is distributed there. The Joule heating around the cathode deaccelerates the plasma, then the expected acceleration near the cathode by the axial Lorentz force can not be obtained.

The argon temperature increases up to about \(10^4\text{K}\) at the cathode tip. The plasma keeps high temperature behind the tip, however the temperature decreases near the flared anode because of the low current density.

The ionization degree approaches about 0.5 at the cathode tip. The ionization degree distribution shows the flow structure of so-called cathode jet, which features both high-temperature and high-density plasma jet emanating from the cathode tip as a result of strong Joule heating.

References