MHD simulation of magnetic reconnection and magnetic substorms

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Magnetic reconnection based on the resistive and Hall MHD equations are reviewed. It is found from both theory and simulation that the impulsive reconnection can be achieved in the resistive and Hall MHD by imposing suitable boundary conditions. All results from resistive and Hall MHD indicate that reconnection processes contain two phases: slow growth phase and impulsive phase, and between them there is a sudden change of growth rate. However, the distribution and magnitude of the parallel electric field in the Hall MHD reconnection are much broader and larger than ones in the resistive MHD reconnection. Furthermore, nonlinear reconnection rate in the Hall MHD is nearly independent in the Lundquist number. The application of the impulsive magnetic reconnection to substorms in the magnetotail is presented.

1. Introduction

The magnetic reconnection as one of the main processes to release the magnetic energy to the kinetic and thermal energy with undergoing the topological change is widely studied. It is believed that the magnetic reconnection is a possible cause of the space phenomena such as the substorms in the magnetotail, flux transfer events at the dayside magnetopause, and solar flare in the solar corona.

The first well-known reconnection model was proposed by Sweet (1958) and Parker (1957). In this model, they suggested that the magnetic reconnection be a steady state process which occurs in the vicinity of the neutral line with a characteristic timescale \( \tau_{SP} \equiv (\tau_A \tau_R)^{1/2} = S^{1/2} \tau_A \), where \( \tau_A = a/V_A = a(4\pi\rho)^{1/2}/B_0 \) is the Alfvén time, \( \tau_R = 4\pi a^2/\eta c^2 \) is the resistive diffusion time, and \( S \equiv \tau_R/\tau_A \) is the Lundquist number, \( \rho \) is the mass density, \( \eta \) is the resistivity of the plasma, and \( c \) is the speed of the light. For the low resistivity or high-S plasmas which are true for the space plasmas, the timescale of the steady state reconnection process is too slow to account for the fast dynamical processes such as solar flares even if \( \tau_{SP} \) is much shorter than the diffusion time \( \tau_R \).

In order to resolve the slow time scale difficulty, Petschek (1964) proposed another steady state model that yields an Alfvénic reconnection rate with a weak dependence on the resistivity. In the Petschek model, the structure of the reconnection is forced to be the X-type geometry from the Y-type geometry in the Sweet-Parker model. The dynamical feasibility of the Petschek model is questionable and the subject of controversy. From numerical simulations (Yan et al., 1993; Ma et al., 1995a), it is suggested that the Petschek-like reconnection scenarios can only be realized in the low-S or high resistive \( (S < 10^3) \) regime. When the Lundquist number is larger than \( 10^4 \), the reconnection always tends to form the Y-type geometry with a slow Sweet-Parker reconnection rate even if boundary condition is imposed to be suitable Petschek-like fast reconnection. The plasmas in both the magnetotail or solar corona have a common feature: the classical Lundquist number is often very high \( (S > 10^{-8}) \). Although the Sweet-Parker-like reconnection is dynamically realizable in the high-S regime, its time scale is too slow. On the other hand, Petschek-like reconnection yields a fast time scale, but dynamically it is impossible in the high-S regime.

2. Time dependent reconnection

The observed dynamical processes in the space plasmas which may be associated with magnetic reconnection are externally driven and strongly time-dependent. This requires that the magnetic reconnection in a model is not steady state and spontaneous. Wang et al. (1996) proposed a time-dependent driven reconnection model with initial Harris sheet condition. In their analytical as well as numerical simulation results, it is found that the peak amplitude of current sheet as shown in Fig. 1 and reconnection electric field exhibits a sudden transition from a slow linear growth phase to a rapid nonlinear phase in the characteristic time scale \( \tau_N = (\tau_R \tau_0 \tau_A^3)^{1/5} \), where \( \tau_0 \) is the characteristic time scale of the imposed boundary flow. It should be noted that the new characteristic time scale \( \tau_N \) has much weaker dependence on the resistivity \( (\eta^{-1/5}) \) than the characteristic time scale in the Sweet-Parker model \( (\eta^{-1/2}) \), but the geometry of the reconnection region retains a Sweet-Parker-like or Y-type structure. Furthermore, current sheet amplitude at the separatrix grows as \( J_0(t/\tau_A)^{3/2} \) in the linear regime, and as \((\tau_R/\tau_0)^{1/2}J_0(t/\tau_A)^{3/2}\) in the nonlinear regime, where \( J_0 \) is...
a constant, independent of $\eta$. The large multiplicative factor $(\tau_R/\tau_0)^{1/2}$ accounts for the suddenness in the enhancement of the current sheet amplitude in the nonlinear regime following a period of sluggish growth.

Ma et al. (1995) conducted a simulation to study magnetotail substorm dynamics starting from a Grad-Shafranov equilibrium of the magnetotail with the Earth dipole field. This initial configuration is driven by imposed electric field at the low and upper boundary. The induced equatorward flow leads to slow development of a thin current sheet in time, with extending from the mid-tail region ($30R_E$) to the near-Earth region ($10R_E$). In the late growth phase, rapid thinning of the current sheet and near-explosive intensification of the current density are observed as shown in Fig. 2. Sudden enhancement of current sheet was also observed by the satellite observations (Ohtani et al., 1992).

3. Hall reconnection

In the previous section, it is shown that the plasmas in the high-S regime tends to develop thin and intensified current sheets in the reconnection layer during the late growth phase in the forced reconnection. As the thickness $\Delta$ of the thin and intense current sheets falls in the range $d_e \equiv c/\omega_{pe} < \Delta < d_i \equiv c/\omega_{pi}$ (where $\omega_{pe}$ and $\omega_{pi}$ are the electron and ion plasma frequencies, respectively), the collisionless or kinetic terms should be retained in the generalized Ohm’s law

$$E + \frac{v \times B}{e} = n J + \frac{4\pi}{\omega_{pe}^2} \frac{D J}{Dt} - \frac{\nabla p}{n e} + \frac{J \times B}{nee}$$

where $E$ is the electric field, $B$ is the magnetic field, $v$ is the plasma flow velocity, $J$ is the plasma current density, $p$ is the electron scalar pressure, $n$ is the electron density, $e$ is the magnitude of the electron charge, $D/J/Dt \equiv \partial \partial t + v \cdot \nabla$ is the total convective derivative. Last two terms on the right hand side of the above generalized Ohm’s law are referred to collectively as Hall MHD effects. The collisionless or Hall MHD terms has widely studied in recent years in the space and fusion plasmas (Aydemir, 1992; Wang and Bhattacharjee, 1993; Cai et al., 1994; Mandt et al., 1994; Biskamp et al.,1997; Ma and Bhattacharjee, 1996; 1998; Shay et al., 1998, Birn et al., 2001; Ma and Bhattacharjee, 2001; Wang et al. 2001).

It is demonstrated in the Harris sheet (Ma and Bhattacharjee, 1996; 2001) that in collisionless plasma, the effects of electron pressure gradients and Hall currents in the magnetic reconnection process can provide not only fast time scale growth but also the impulsiveness with a very weakly dependence of the resistivity. The structure of thin current sheets during the dynamical evolution of such Hall MHD plasmas changes naturally from a Sweet-Parker-like or Y-type geometry in the early stage to a Petschek-like or X-type geometry as shown in Fig. 3. However, the underlying physics of reconnection is qualitatively different from that in resistive MHD models. In Hall MHD, the electron and ion dynamics are decoupled from each other in the reconnection region. Such decoupling is forbidden in the resistive MHD. In Hall MHD the spatial scale of the electric field, which is the ion skin depth, is much broader than the spatial scale of current density, which is determined by the resistivity or
Fig. 4. Time history of the cross tail current density at a typical near-Earth distance \((x, z) = (-12, 0)\) with different resistivity.

the electron skin depth, while electric field in resistive MHD has the same spatial scale as current density. The results from Hall MHD simulation with initial magnetotail configuration (Ma and Bhattacharjee, 1998) show that the dynamics of thin current sheet in the linear and early nonlinear phase is qualitatively similar to that observed in the resistive MHD simulation (Ma et al. 1995b), but there are significant differences in the impulsive growth stage. Two distinctive features emerge from the Hall MHD simulation. First, the amplitude of thin current sheet at the near-Earth region, after a period of slow growth in time, exhibits a sudden impulsive enhancement at a sub-Alfvénic growth rate that is insensitive to the value of resistivity as shown in Figure 4. Second, the current sheet is thinner and shows a greater dynamic variability than in the resistive MHD simulation. Figure 5 shows the time evolution of the cross-tail current density and magnetic field topology during substorm period. It is evident from Fig. 5 that the extended Y-type structure of current sheet seen in resistive MHD (Ma et al. 1995b) shrinks substantially, penetrates to near-earth distances in the slow and impulsive phases, and subsequently propagates tailward after current disruption. In this Hall MHD simulation, the time evolution of the cross tail current density during the whole substorm phase is good agreement with the satellite observations (Ohtani et al., 1992).

In summary, the force reconnection in resistive MHD does give a fast time and shows an impulsive signature but the electric field is too weak to account for the heating and acceleration of electrons in space plasmas. With including Hall MHD effects, the results shows not only fast and explosive reconnection rate but also strong electric field. These crucial phenomena make a strong argument that the Hall MHD effects in the space plasmas are very important at a fundamental level in reconnection dynamics.

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References


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